



UNIVERSITY OF ILLINOIS  
AT URBANA-CHAMPAIGN

# AC Measurement of Magnetic Susceptibility. Part 2.

**Physics 401, Spring 2013**  
**Eugene V. Colla**



# Outline

- **What and how we measuring (week1)**
- **Magnetic losses**
- **Temperature dependencies of permeability**
- **End of semester**



# Magnetic materials.

$$B = \mu_0 (H + M)$$

Magnetic induction

Permeability  
of free space

Magnetic field

Magnetization

$$M = \chi \cdot H$$

$\chi$  – magnetic susceptibility



# Magnetic materials.

$$B = \mu_0 (H + M)$$

$$M = \chi \cdot H$$

In general  $\chi$  is a function of  $H$  and  $T$

$$M = \chi(H, T) \cdot H$$

$$\chi(H, T) = \left( \frac{\partial M}{\partial H} \right)_T$$



# Magnetic materials.

$$B = \mu_0 (H + \chi H) = \mu_0 (1 + \chi) H = \mu_0 \mu_r H;$$

$$\mu_r = 1 + \chi$$

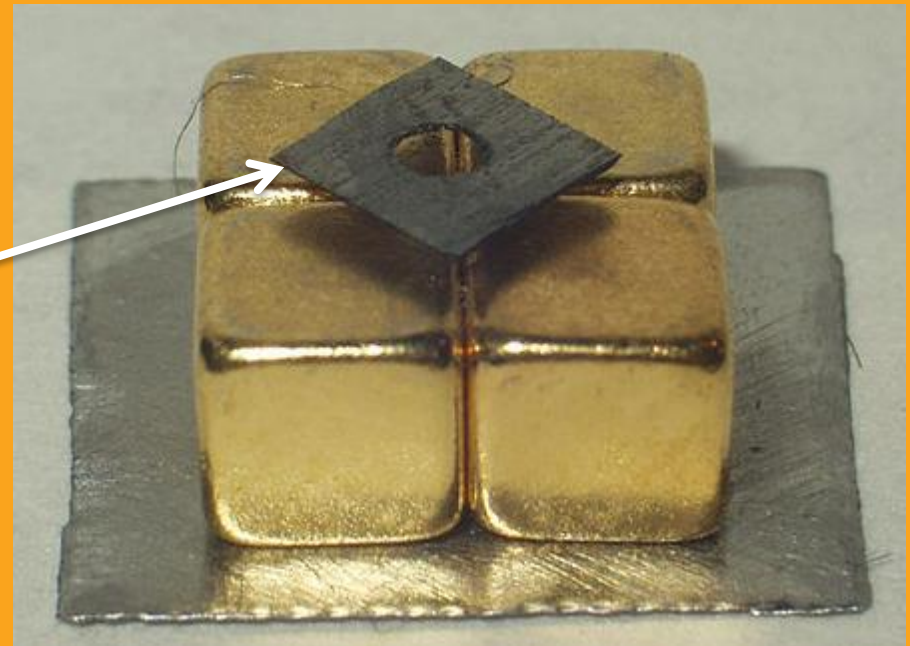
$\chi < 0$	-	diamagnetics,	$\mu_r < 1$
$\chi > 0$	-	paramagnetics	$\mu_r > 1$
$\chi \gg 0$	-	ferromagnetics	$\mu_r \gg 1$



# Magnetic materials. Diamagnetism.

Material	$\chi_v$ ( $10^{-5}$ )
<a href="#">Bismuth</a>	-16.6
<a href="#">Carbon (diamond)</a>	-2.1
<a href="#">Carbon (graphite)</a>	-1.6
<a href="#">Copper</a>	-1.0
<a href="#">Lead</a>	-1.8
<a href="#">Mercury</a>	-2.9
<a href="#">Pyrolytic carbon</a>	<b>-40.0</b>
<a href="#">Silver</a>	-2.6
Superconductor	$-10^5$
<a href="#">Water</a>	-0.91

$\chi < 0$  - diamagnetics



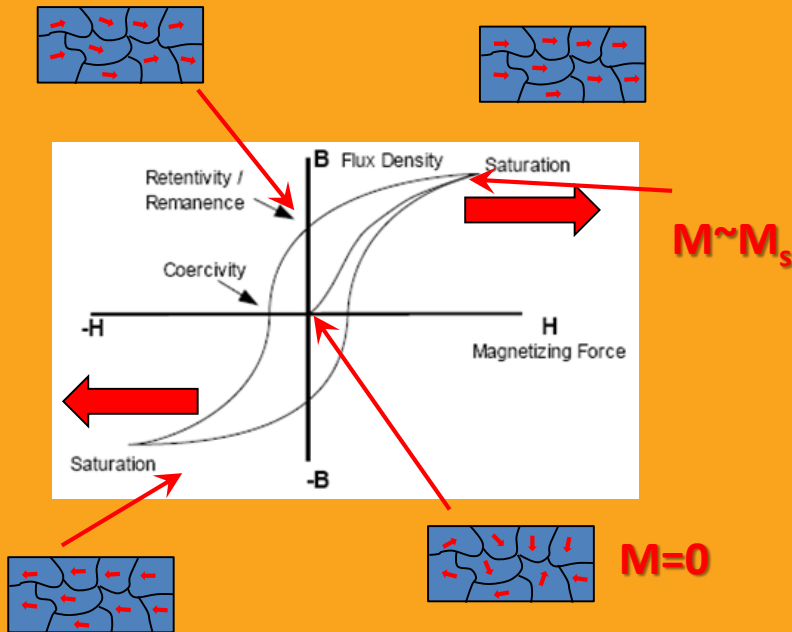
Levitation of the pyrolytic carbon

Ideal diamagnetic  $\chi = -1$

Courtesy of Wikipedia

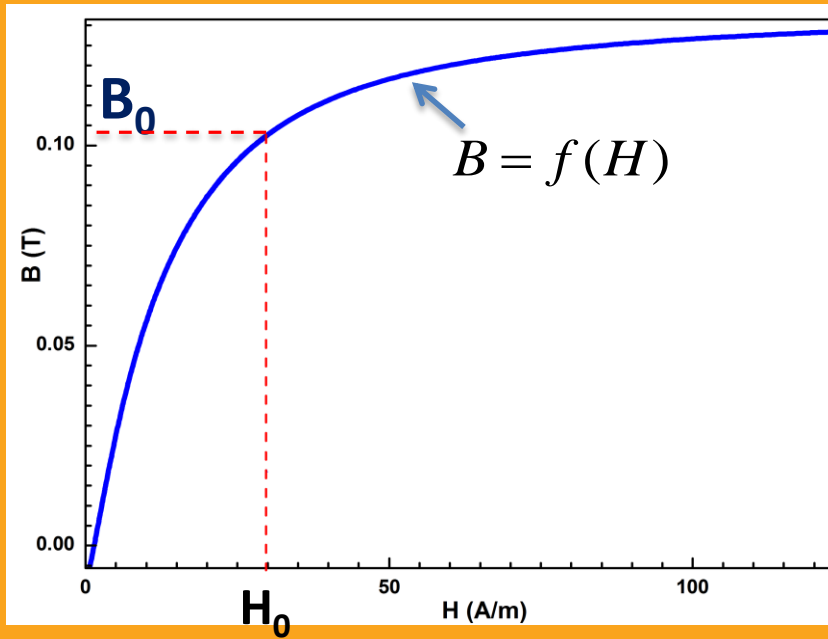
# Magnetic materials. Ferromagnetism.

$\chi \gg 1$  - ferromagnetics

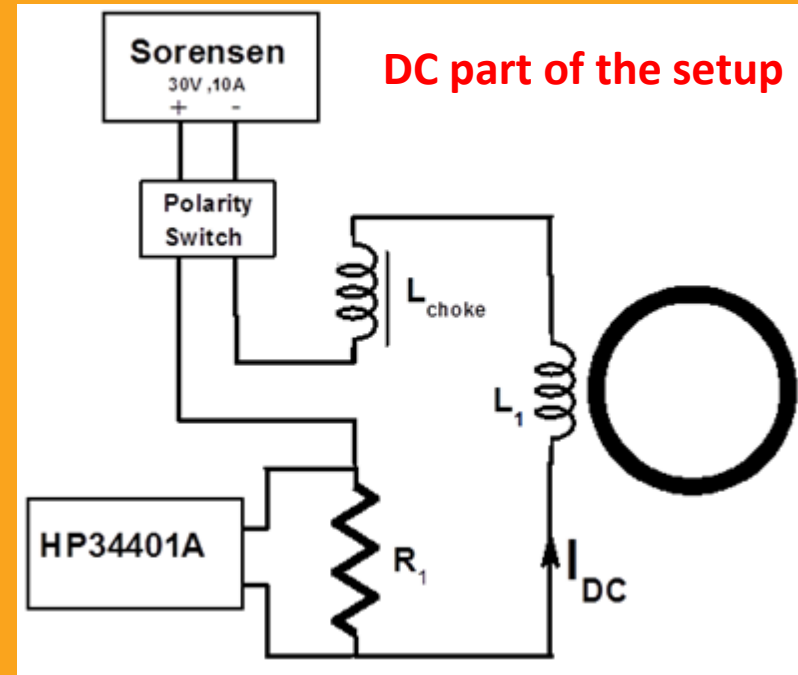


Material	$\mu_r$	$B_{rem}$ (T)
Fe, 99.8% pure	5000	1.3
Permalloy	100,000	0.7
Superpermalloy	1,000,000	0.7
Co, 99% pure	250	0.5
Ni, 99% pure	600	0.4

# Measuring the permeability. DC field.



$$H = H_0 + H_1 \sin \omega t$$

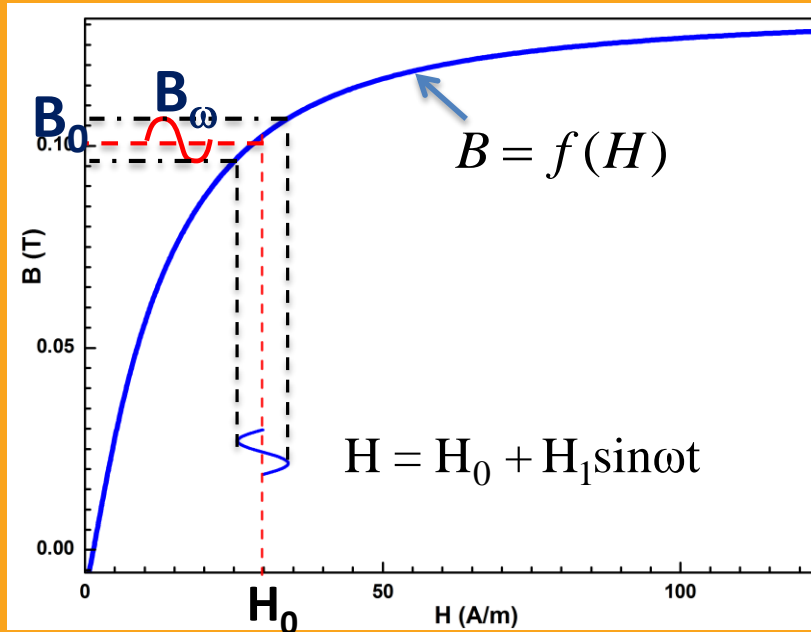


$$H_0 = \frac{N_p I_{DC}}{2\pi r}$$

Here  $N_p$  – number of turns in DC primary coil

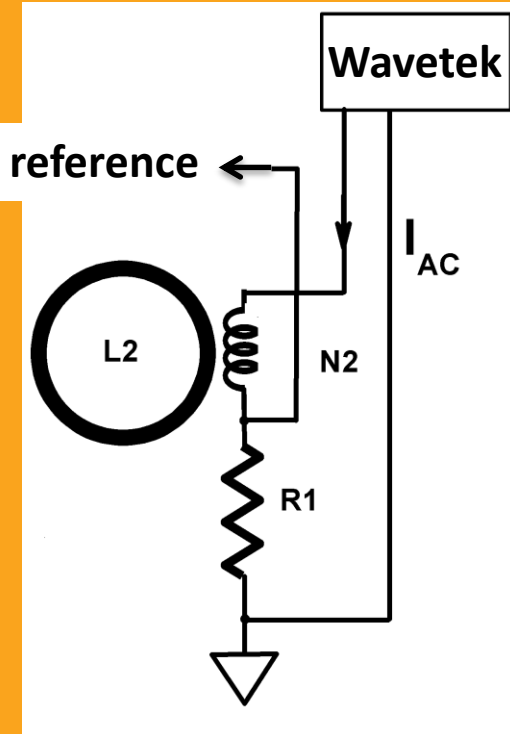
# Measuring of the permeability. AC modulation

$$H = H_0 + H_1 \sin \omega t$$



$$H_1 = \frac{N_p I_{AC}}{2\pi r}$$

$$B_\omega \sim \frac{df}{dH} = \frac{dB}{dH} = \mu = \mu_0 \mu_r$$

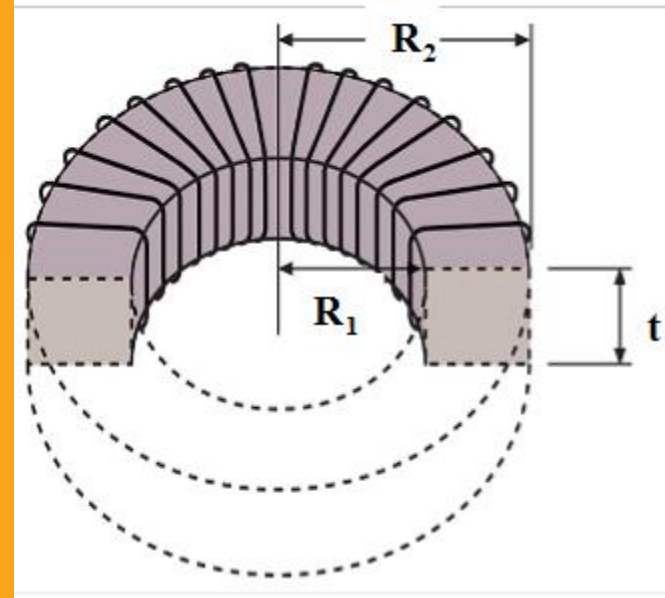


Here  $N_p$  – number of turns in AC primary coil

# Measuring of the permeability. AC modulation

Primary coil of  $N_p$  turns supplied by current  $I_p$  creates magnetic field  $H$  and flux  $\Phi$

$$\text{For toroid: } H = \frac{N_p I_p}{2\pi r}$$



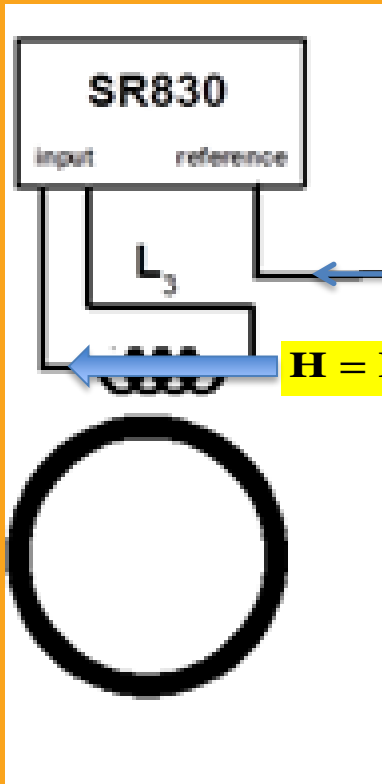
$$\Phi = \mu \int \vec{H} \cdot d\vec{a} = \frac{\mu I N t}{2\pi} \int_{R_1}^{R_2} \frac{dr}{r} = \frac{\mu I N t}{2\pi} \ln \frac{R_2}{R_1}$$

$$R_2 < r < R_1$$

$$da = dr * t$$

# Measuring of the permeability. Pickup coil.

Lock-in measures emf on the pickup coil



$$V_{lock-in} = -N_{pickup} \frac{d\Phi}{dt}$$

Faraday's law

$$\Phi = \mu \int \vec{H} \cdot d\vec{a} = \frac{\mu I_{AC} N t}{2\pi} \int_{R_1}^{R_2} \frac{dr}{r} = \frac{\mu I N t}{2\pi} \ln \frac{R_2}{R_1}$$

This is per 1 turn of the pickup coil !

Here  $I_p$  is ac current in primary coil L3;  $I_p = \frac{V_0 \sin(\omega t)}{R_2}$

$$V_{lock-in} = N_{pickup} \frac{\mu N t}{2\pi} \ln \frac{R_2}{R_1} \frac{dI_{AC}}{dt} = N_{pickup} \frac{\mu N t}{2\pi} \ln \frac{R_2}{R_1} \frac{V_{AC}}{R_1} \omega \cos(\omega t) =$$

$$= \mu_r L_0 \frac{V_{AC}}{R_1} \omega \cos(\omega t);$$

$$\text{where } L_0 = N_{pickup} \frac{\mu_0 N t}{2\pi} \ln \frac{R_2}{R_1}$$



# Measuring of the permeability. Pickup coil.

$$V_{lock-in} = N_{pickup} \frac{\mu N t}{2\pi} \ln \frac{R_2}{R_1} \frac{dI_{AC}}{dt} = N_{pickup} \frac{\mu N t}{2\pi} \ln \frac{R_2}{R_1} \frac{V_{AC}}{R_1} \omega \cos(\omega t) =$$

$$= \mu_r L_0 \frac{V_{AC}}{R_1} \omega \cos(\omega t); \quad \text{where } L_0 = N_{pickup} \frac{\mu_0 N t}{2\pi} \ln \frac{R_2}{R_1}$$

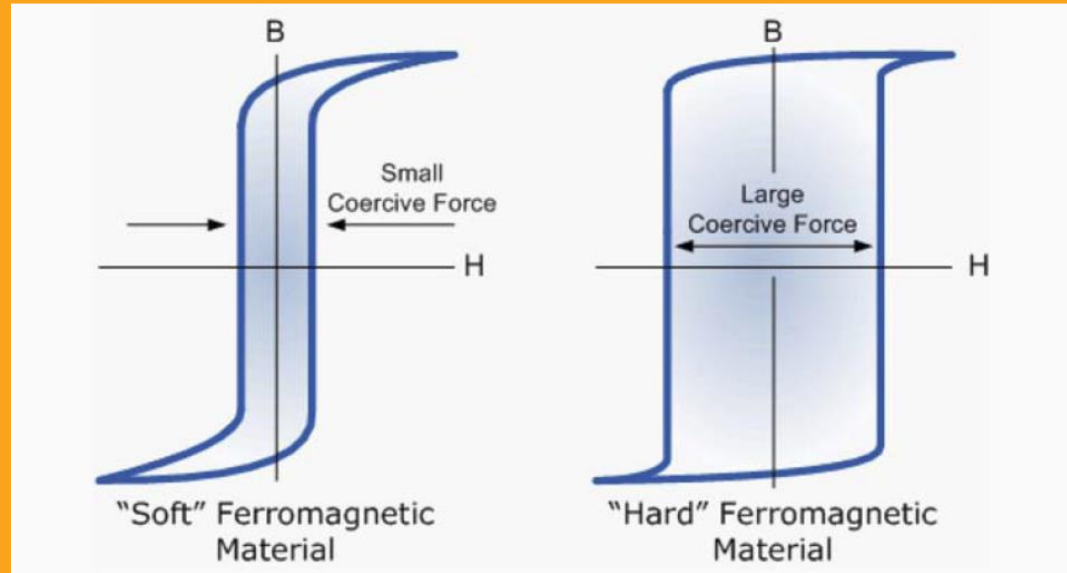
In general  $\mu_r = \mu' - j\mu''$

Geometry  
of toroid

Resistor in  
AC current  
loop

Because of  $\pi/2$  phase shift ( $\cos(\omega t)$ ) the " $\mu''$ " signal will be delivered to Y channel of the lock-in amplifier.  $\mu''$  provides the information about the losses in system

# Hysteresis Loops. Remagnetization loses



Energy of the magnetic field

$$W = V \int H dB$$

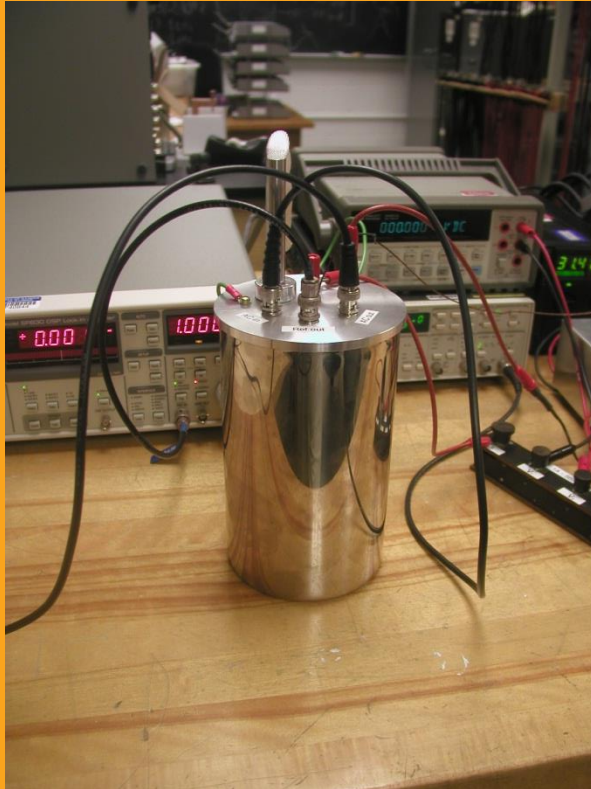
By cycling around the loop

$$W_{loop} = V \oint H dB = V * Loop\_area$$

V here is a volume of the magnetic material

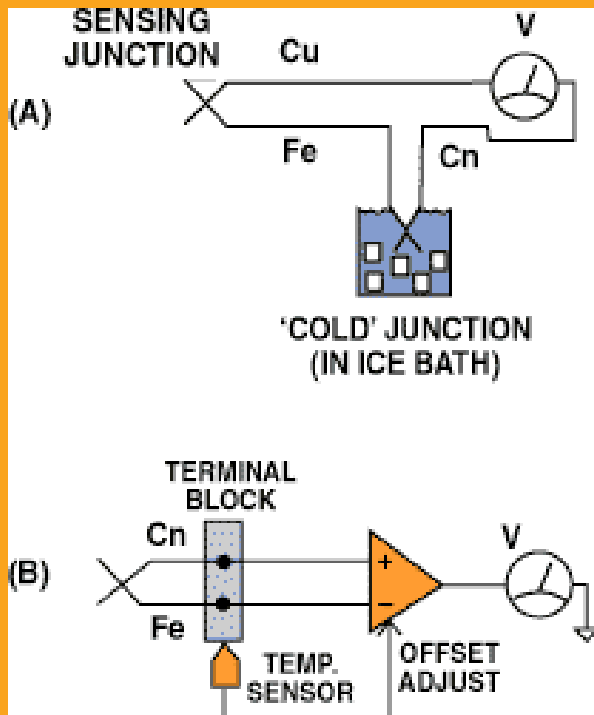


# Temperature dependence of the magnetic permeability.



Now the experiment measurement is going with fixed  $I_{DC}$  and DMM measures the emf of T-type thermocouple

# Measuring of the temperature. Thermocouple.

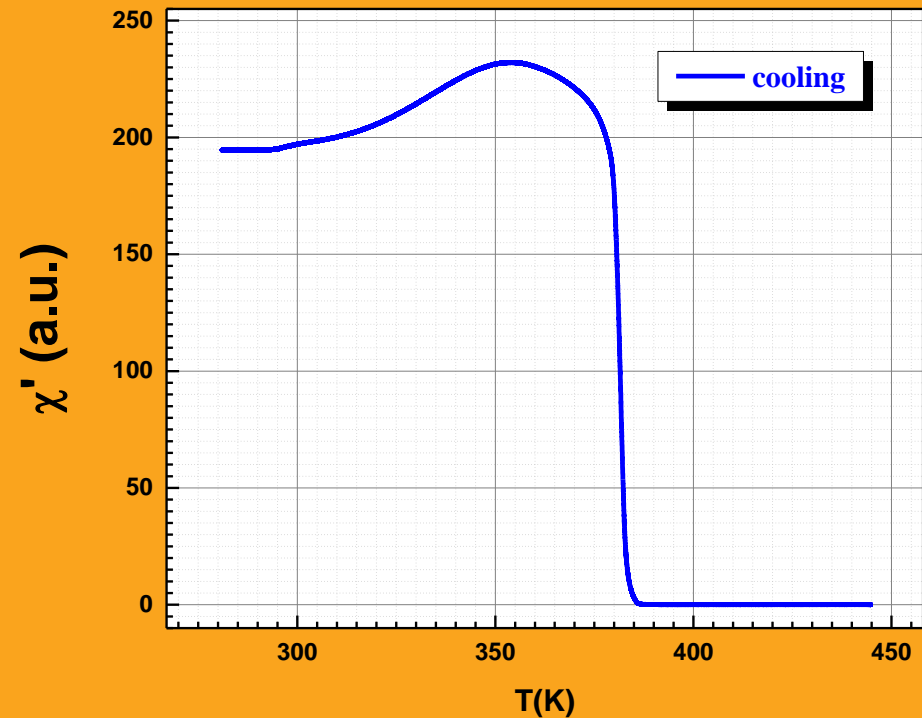
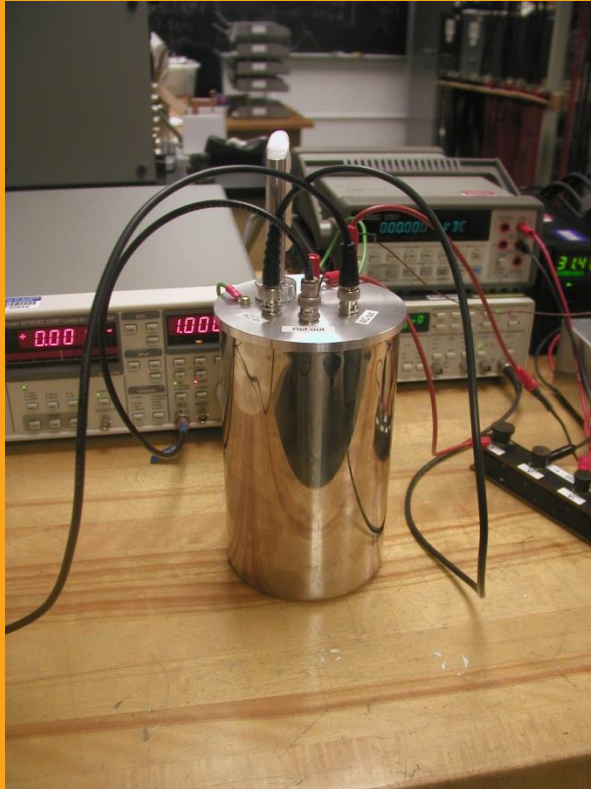


Type	Names of Materials	T Range
B	Platinum 30% Rhodium (+) Platinum 6% Rhodium (-)	2500 -3100F 1370-1700C
C	W5Re Tungsten 5% Rhenium (+) W26Re Tungsten 26% Rhenium (-)	3000-4200F 1650-2315C
E	Chromel (+) Constantan (-)	200-1650F 95-900C
J	Iron (+) Constantan (-)	200-1400F 95-760C
K	Chromel (+) Alumel (-)	200-2300F 95-1260C
N	Nicrosil (+) Nisil (-)	1200-2300F 650-1260C
R	Platinum 13% Rhodium (+) Platinum (-)	1600-2640F 870-1450C
S	Platinum 10% Rhodium (+) Platinum (-)	1800-2640F 980-1450C
T	<b>Copper (+) Constantan (-)</b>	<b>-330-660F -200-350C</b>

Type T (copper-constantan) has thermoemf at 0°C 41.5μV/°C;

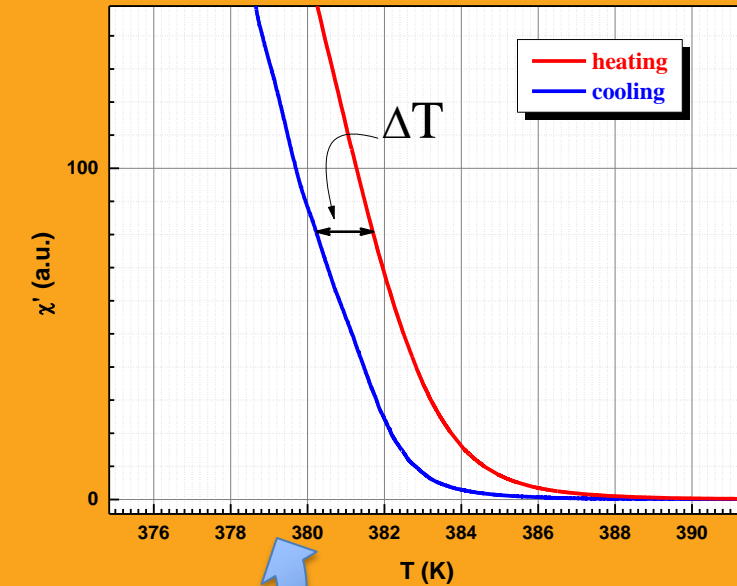
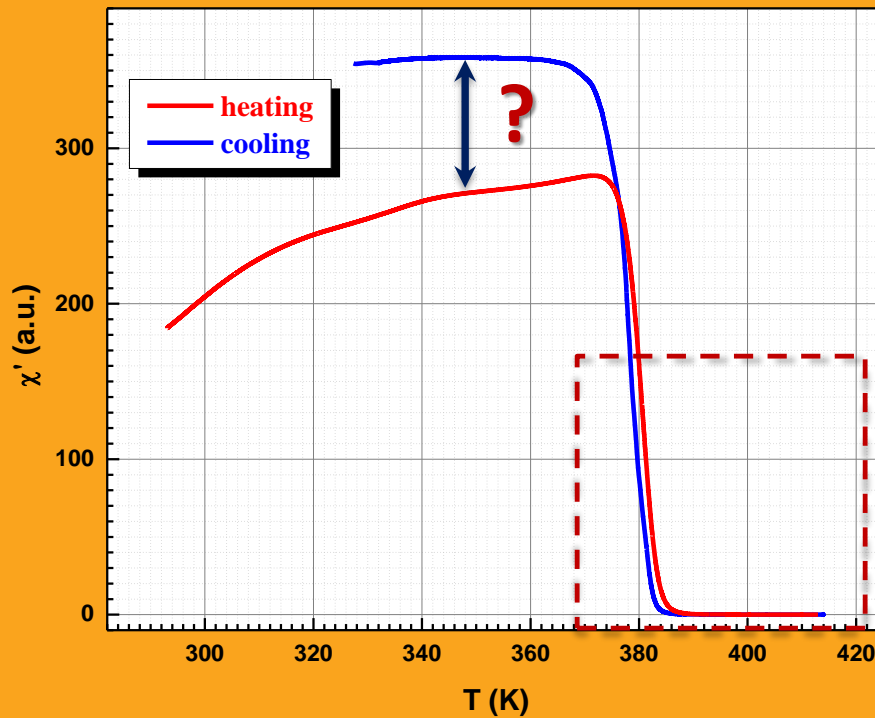
# Temperature dependence of the magnetic permeability.

## Ferroxcube 3E8



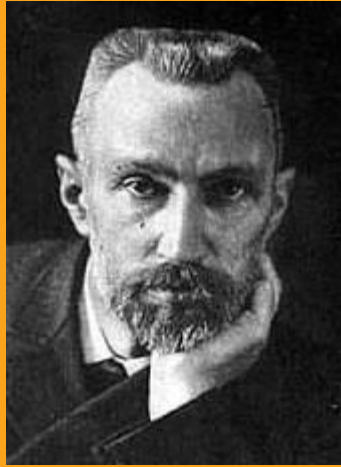
# Temperature dependence of the magnetic permeability.

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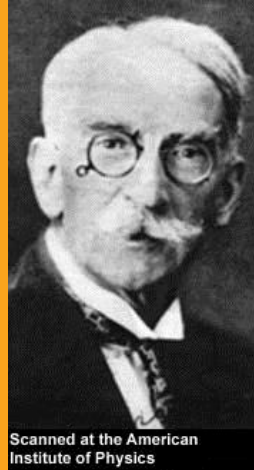


Hysteresis. Where it is coming from?

# Temperature dependence of the magnetic permeability. Curie-Weiss law.

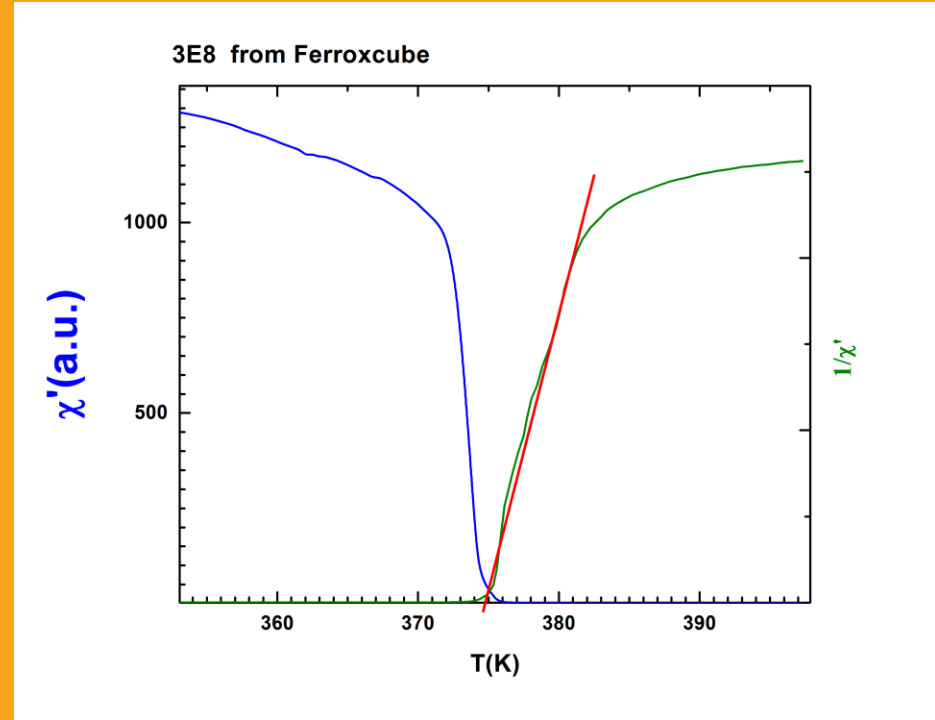


**Pierre Curie**  
5.15.1859-4.19.1906



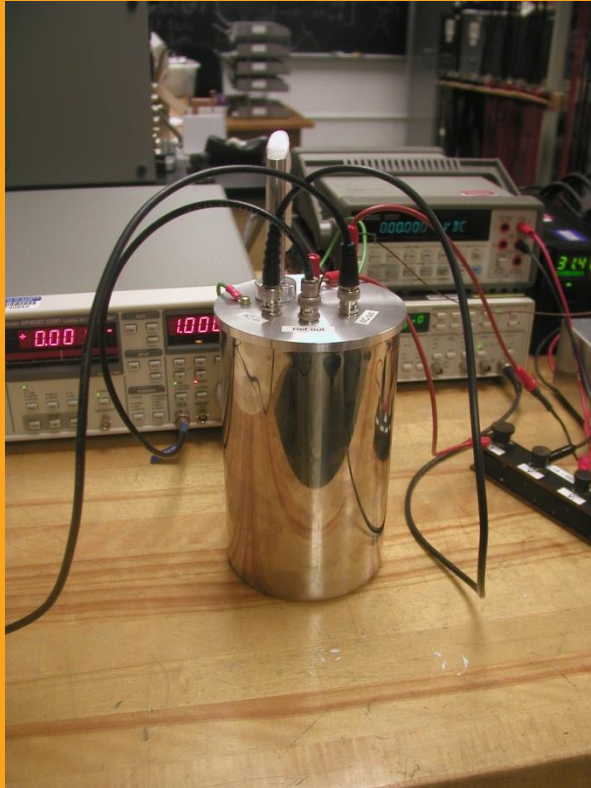
**Pierre Ernest Weiss**  
3.25.1865-10.24.1940

$$\chi' = \frac{C}{T - T_c}$$

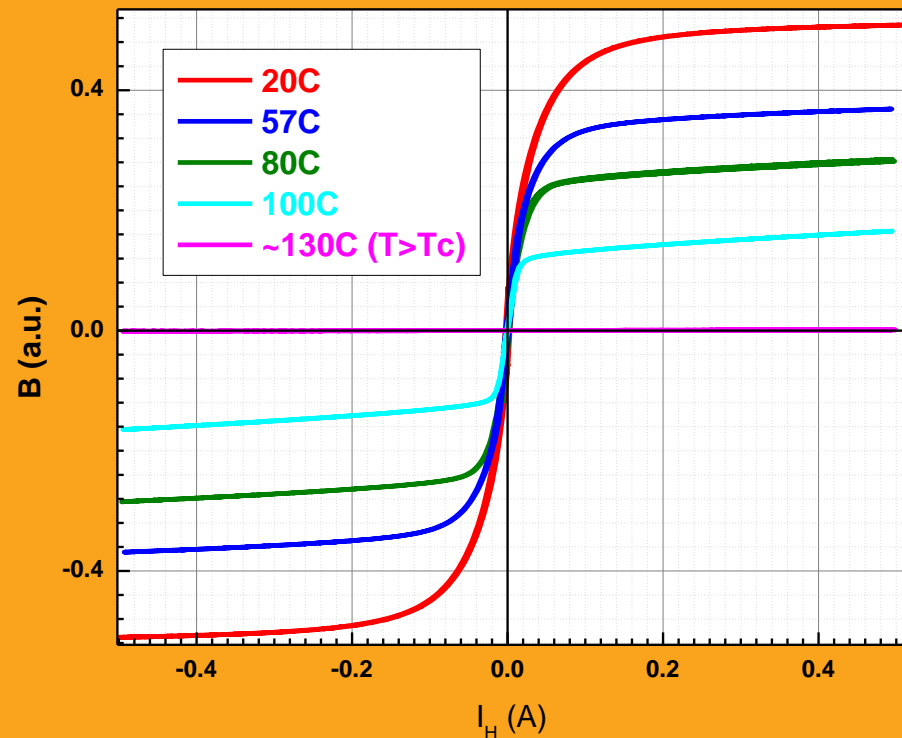


**Curie-Weiss law**

# Temperature dependence of the magnetic permeability



## Ferroxcube 3E8



# References

- **Information about magnetic materials can be found in :**  
**\\Phyapportal\PHYCS401\Experiments\AC\_Magnetization\Magnetic Materials**
- **Chapter from book by D.J. Craik and R.S. Tebble**  
**“Ferromagnetism and Ferromagnetic domains” link on P401 home page**
- **SR830 manual *ibid***



# End of semester schedule

- **No Lecture on 22<sup>nd</sup> April**
- **Third week of the Magnetic Laboratory. You can repeat some experiments done during first two weeks + some new challenging experiments like taking B-H dependencies taken at different temperatures or measuring of the permeability as a function frequency with and without DC magnetic field bias.**
- **May 7<sup>th</sup> Tuesday 11.59pm. Final deadline for submitting of the final report. No extension and no late vouchers.**

